Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

11 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0, 8)$ and maximum point $(6, 16)$ as shown in the following diagram.
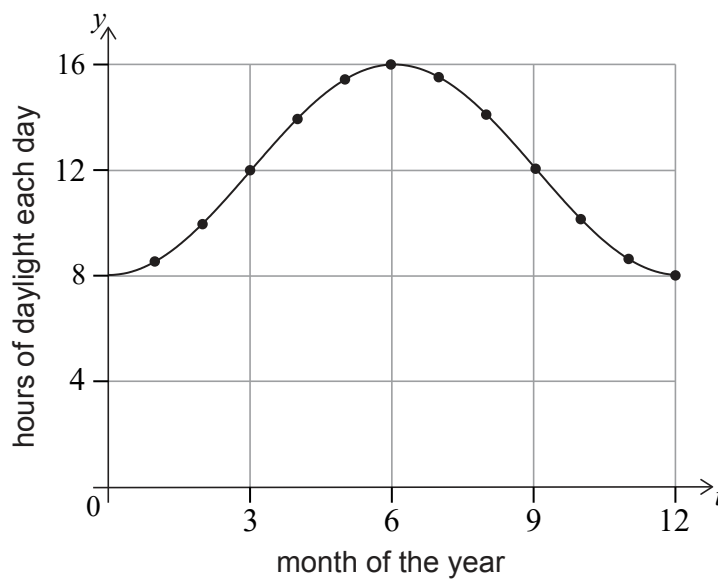
Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

(a)   Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years.   [1]

(This question continues on the following page)

(Question 1 continued)

Paula thinks that a better model is $f(t) = a\cos(bt) + d$, $t \geq 0$, for specific values of a, b and d.

(b)   For Paula's model, use the diagram to write down

      (i)   the amplitude.

      (ii)   the period.

      (iii)   the equation of the principal axis.   [4]

(c)   Hence or otherwise find the equation of this model in the form: [3]

$$f(t) = a\cos(bt) + d$$

(d)   For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day.   [4]

The true maximum number of daylight hours was 16 hours and 14 minutes.

(e)   Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram.   [3]

Turn over

2.   [Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\,\text{cm}$ wide and $20\,\text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.

diagram not to scale

$n = 1$

tile

$l_1 = 30$

$w_1 = 20$

$n = 3$

tile

$l_3 = 50$

$w_3 = 40$
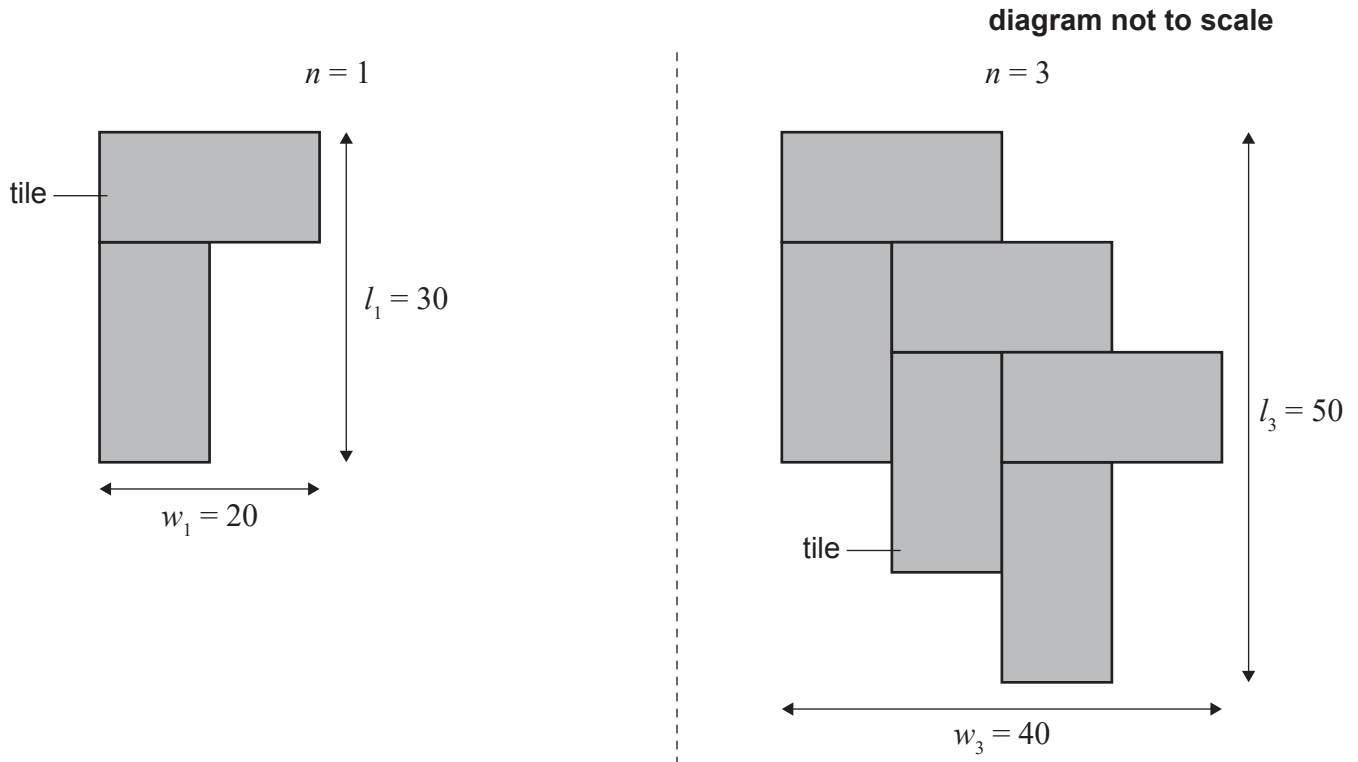
The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(This question continues on the following page)

(Question 2 continued)

(a)   Find the value of

  (i) a.

  (ii) b.   [2]

(b)   Write down an expression in terms of n for

  (i) w_n.

  (ii) l_n.   [3]

Eddie's lawn has a length $740\,\text{cm}$.

(c)   (i)   Show that Eddie needs 144 tiles.

  (ii)   Find the value of w_n for this path.   [3]

(d)   Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer.   [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e)   Find the cost of a single pack of five tiles.   [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f)   Find the minimum number of packs of tiles Eddie will need to order.   [3]

There is a fixed delivery cost of $\$35$.

(g)   Find the total cost for Eddie's order.   [2]

Turn over

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3.   [Maximum mark: 16]

The scores of the eight highest scoring countries in the 2019 Eurovision song contest are shown in the following table.

	Eurovision score
Netherlands	498
Italy	472
Russia	370
Switzerland	364
Sweden	334
Norway	331
North Macedonia	305
Azerbaijan	302

(a)   For this data, find

    (i)   the upper quartile.

    (ii)   the interquartile range.   [4]

(b)   Determine if the Netherlands' score is an outlier for this data. Justify your answer.   [3]

(This question continues on the following page)

Turn over

(Question 3 continued)

Chester is investigating the relationship between the highest-scoring countries' Eurovision score and their population size to determine whether population size can reasonably be used to predict a country's score.

The populations of the countries, to the nearest million, are shown in the table.

	Population (x) (millions)	Eurovision score (y)
Netherlands	17	498
Italy	60	472
Russia	145	370
Switzerland	9	364
Sweden	10	334
Norway	5	331
North Macedonia	2	305
Azerbaijan	10	302

Chester finds that, for this data, the Pearson's product moment correlation coefficient is $r = 0.249$.

(c) State whether it would be appropriate for Chester to use the equation of a regression line for y on x to predict a country's Eurovision score. Justify your answer. [2]

Chester then decides to find the Spearman's rank correlation coefficient for this data, and creates a table of ranks.

	Population rank (to the nearest million)	Eurovision score rank
Netherlands	3	1
Italy	2	2
Russia	1	3
Switzerland	a	4
Sweden	b	5
Norway	7	6
North Macedonia	8	7
Azerbaijan	c	8

(This question continues on the following page)

(Question 3 continued)

    (d)   Write down the value of:

        (i) a,

        (ii) b,

        (iii) c.   [3]

    (e)   (i)   Find the value of the Spearman's rank correlation coefficient r_s.

        (ii)   Interpret the value obtained for r_s.   [3]

    (f)   When calculating the ranks, Chester incorrectly read the Netherlands' score as 478. Explain why the value of the Spearman's rank correlation r_s does not change despite this error.   [1]
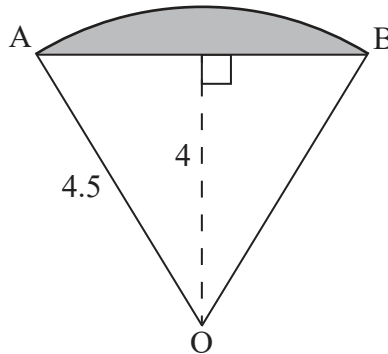
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4.   [Maximum mark: 15]

A sector of a circle, centre O and radius $4.5\,\text{m}$, is shown in the following diagram.

diagram not to scale

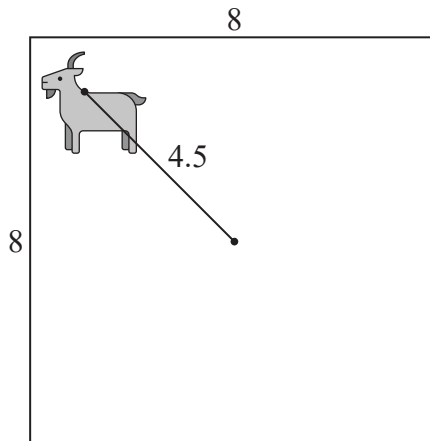(a)   (i)   Find the angle $\hat{\text{AOB}}$.

(ii)   Find the area of the shaded segment.   [8]

A square field with side $8\,\text{m}$ has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to $4.5\,\text{m}$ from the post.

diagram not to scale

(b)   (i)   Find the area of a circle with radius $4.5\,\text{m}$.

(ii)   Find the area of the field that can be reached by the goat.   [5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 0.3\,t\mathrm{e}^{-t}$.

(c)   Find the value of t at which the goat is eating grass at the greatest rate.   [2]

5. [Maximum mark: 15]

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

(a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]

(b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]

(ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays $\$150$ for a ticket. If too many passengers arrive, then the airline will give $\$300$ in compensation to each passenger that cannot board.

(c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

References:

2. mynamepong, n.d. Goat [image online] Available at: https://thenounproject.com/term/goat/1761571/
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